

HONORS COLLEGE ALG

TRANSFORMATIONS + PIECE FUNC.

Exercises

Name Key
Period —

In Problems 7–18, match each graph to one of the following functions.

A. $y = x^2 + 2$

B. $y = -x^2 + 2$

C. $y = |x| + 2$

D. $y = -|x| + 2$

E. $y = (x - 2)^2$

F. $y = -(x + 2)^2$

G. $y = |x - 2|$

H. $y = -|x + 2|$

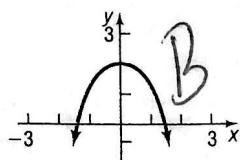
I. $y = 2x^2$

J. $y = -2x^2$

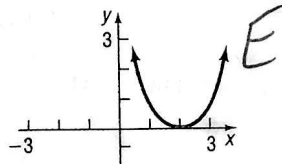
K. $y = 2|x|$

L. $y = -2|x|$

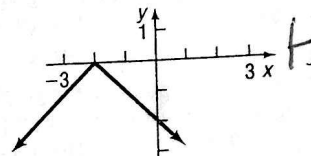
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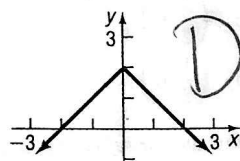
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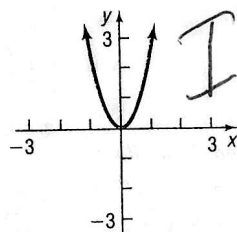
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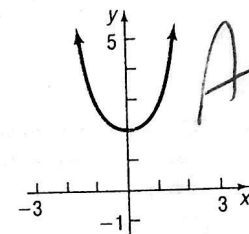
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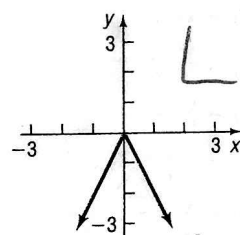
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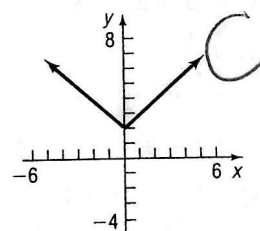
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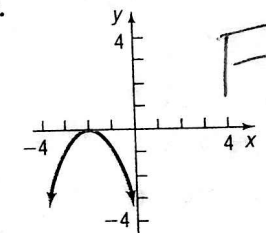
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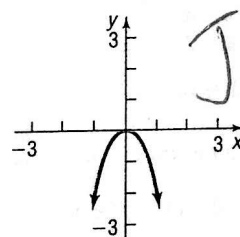
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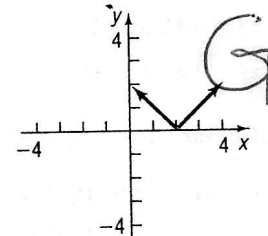
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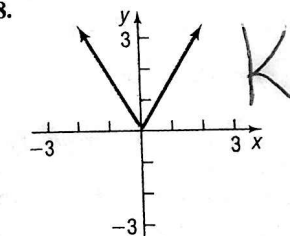
16.



17.



18.



In Problems 19–26, write the function whose graph is the graph of $y = x^3$, but is:

19. Shifted to the right 4 units $y = (x - 4)^3$

21. Shifted up 4 units $y = x^3 + 4$

23. Reflected about the y-axis $y = -x^3$

25. Vertically stretched by a factor of 4 $y = 4x^3$

20. Shifted to the left 4 units $y = (x + 4)^3$

22. Shifted down 4 units $y = x^3 - 4$

24. Reflected about the x-axis $y = -x^3$

26. Horizontally stretched by a factor of 4

In Problems 27–30, find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$.

27. (1) Shift up 2 units $y = \sqrt{x} + 2$
(2) Reflect about the x-axis $y = -(\sqrt{x} + 2)$
(3) Reflect about the y-axis $y = -(\sqrt{-x} + 2)$

29. (1) Reflect about the x-axis $y = -\sqrt{x}$
(2) Shift up 2 units $y = -\sqrt{x} + 2$
(3) Shift left 3 units $y = -\sqrt{x + 3} + 2$

28. (1) Reflect about the x-axis $y = -\sqrt{x}$
(2) Shift right 3 units $y = -\sqrt{x - 3}$
(3) Shift down 2 units $y = -\sqrt{x - 3} - 2$

30. (1) Shift up 2 units $y = \sqrt{x} + 2$
(2) Reflect about the y-axis $y = \sqrt{-x} + 2$
(3) Shift left 3 units $y = \sqrt{-x - 3} + 2$

31. If $(3, 0)$ is a point on the graph of $y = f(x)$, which of the following must be on the graph of $y = -f(x)$?

- (a) $(0, 3)$ (b) $(0, -3)$
(c) $(3, 0)$ (d) $(-3, 0)$

33. If $(0, 3)$ is a point on the graph of $y = f(x)$, which of the following must be on the graph of $y = 2f(x)$?

- (a) $(0, 3)$ (b) $(0, 2)$
(c) $(0, 6)$ (d) $(6, 0)$

32. If $(3, 0)$ is a point on the graph of $y = f(x)$, which of the following must be on the graph of $y = f(-x)$?

- (a) $(0, 3)$ (b) $(0, -3)$
(c) $(3, 0)$ (d) $(-3, 0)$

34. If $(3, 0)$ is a point on the graph of $y = f(x)$, which of the following must be on the graph of $y = \frac{1}{2}f(x)$?

- (a) $(3, 0)$ (b) $(\frac{3}{2}, 0)$
(c) $(0, \frac{3}{2})$ (d) $(\frac{1}{2}, 0)$

In Problems 35–64, graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function (for example, $y = x^2$) and show all stages.

35. $f(x) = x^2 - 1$

36. $f(x) = x^2 + 4$

37. $g(x) = x^3 + 1$

38. $g(x) = x^3 - 1$

39. $h(x) = \sqrt{x - 2}$

40. $h(x) = \sqrt{x + 1}$

41. $f(x) = (x - 1)^3 + 2$

42. $f(x) = (x + 2)^3 - 3$

43. $g(x) = 4\sqrt{x}$

44. $g(x) = \frac{1}{2}\sqrt{x}$

45. $h(x) = \frac{1}{2x}$

46. $h(x) = 3\sqrt[3]{x}$

47. $f(x) = -\sqrt[3]{x}$

48. $f(x) = -\sqrt{x}$

49. $g(x) = |-x|$

50. $g(x) = \sqrt[3]{-x}$

51. $h(x) = -x^3 + 2$

52. $h(x) = \frac{1}{-x} + 2$

53. $f(x) = 2(x + 1)^2 - 3$

54. $f(x) = 3(x - 2)^2 + 1$

55. $g(x) = \sqrt{x - 2} + 1$

56. $g(x) = |x + 1| - 3$

57. $h(x) = \sqrt{-x} - 2$

58. $h(x) = \frac{4}{x} + 2$

59. $f(x) = -(x + 1)^3 - 1$

60. $f(x) = -4\sqrt{x - 1}$

61. $g(x) = 2|1 - x|$

62. $g(x) = 4\sqrt{2 - x}$

63. $h(x) = 2 \operatorname{int}(x - 1)$

64. $h(x) = \operatorname{int}(-x)$

In Problems 65–70, the graph of a function f is illustrated. Use the graph of f as the first step toward graphing each of the following functions:

(a) $F(x) = f(x) + 3$

(b) $G(x) = f(x + 2)$

(c) $P(x) = -f(x)$

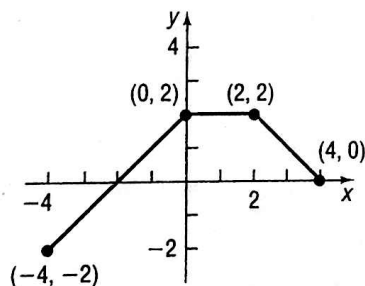
(d) $H(x) = f(x + 1) - 2$

(e) $Q(x) = \frac{1}{2}f(x)$

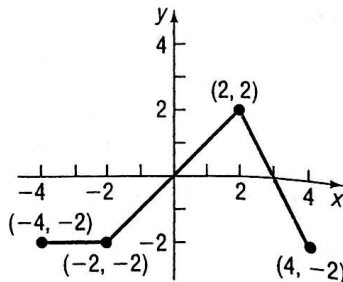
(f) $g(x) = f(-x)$

(g) $h(x) = f(2x)$

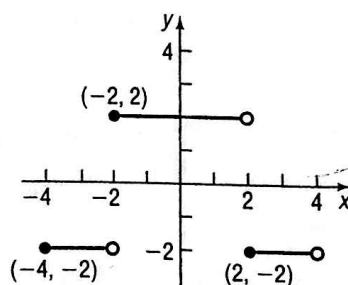
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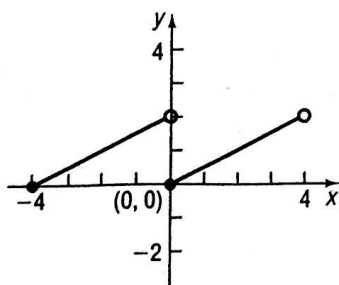
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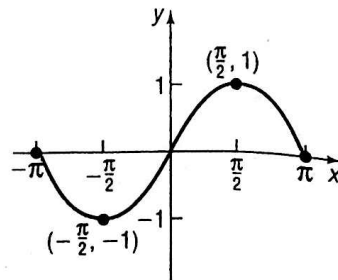
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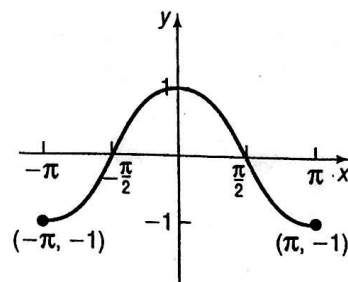
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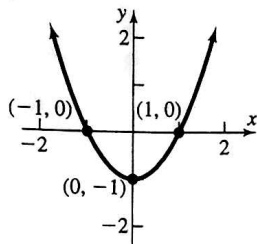
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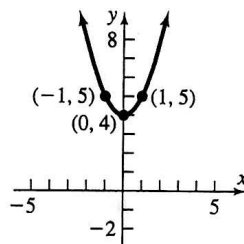
3.5 Exercises (page 272)

7. B 8. E 9. H 10. D 11. I 12. A 13. L 14. C 15. F 16. J 17. G 18. K 19. $y = (x - 4)^3$ 20. $y = (x + 4)^3$
 21. $y = x^3 + 4$ 22. $y = x^3 - 4$ 23. $y = -x^3$ 24. $y = -x^3$ 25. $y = 4x^3$ 26. $y = \left(\frac{1}{4}x\right)^3 = \frac{1}{64}x^3$
 27. (1) $y = \sqrt{x} + 2$; (2) $y = -(\sqrt{x} + 2)$; (3) $y = -(\sqrt{-x} + 2)$ 28. (1) $y = -\sqrt{x}$; (2) $y = -\sqrt{x - 3}$; (3) $y = -\sqrt{x - 3} - 2$
 29. (1) $y = -\sqrt{x}$; (2) $y = -\sqrt{x} + 2$; (3) $y = -\sqrt{x + 3} + 2$ 30. (1) $y = \sqrt{x} + 2$; (2) $y = \sqrt{-x} + 2$; (3) $y = \sqrt{-x - 3} + 2$
 31. (c) 32. (d) 33. (c) 34. (a)

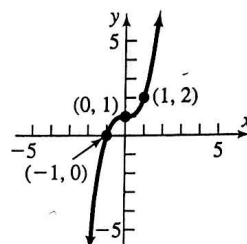
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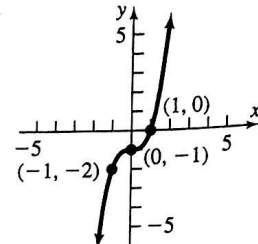
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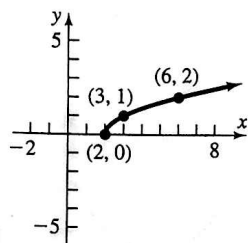
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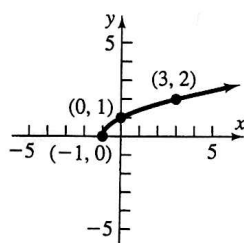
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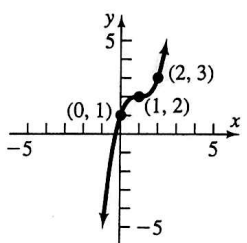
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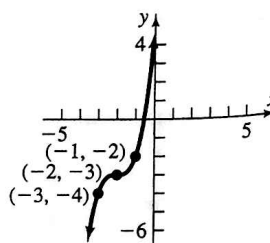
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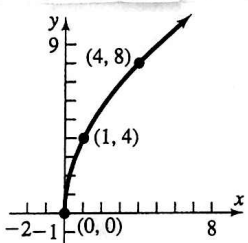
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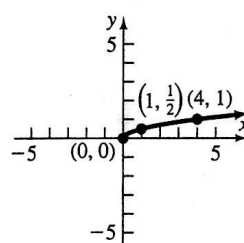
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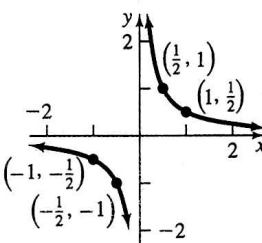
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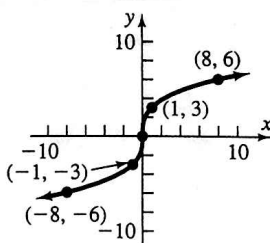
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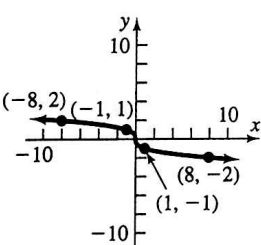
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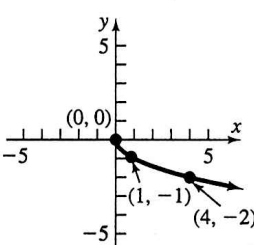
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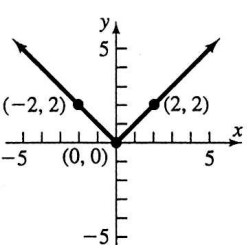
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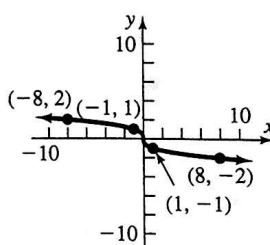
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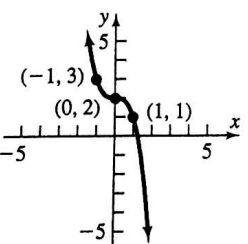
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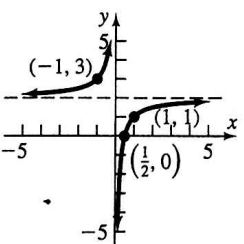
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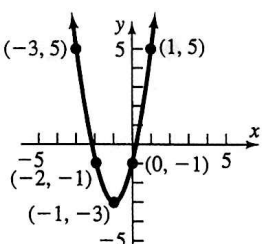
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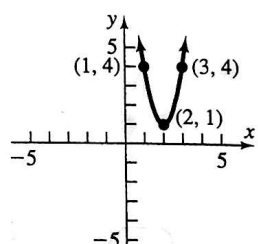
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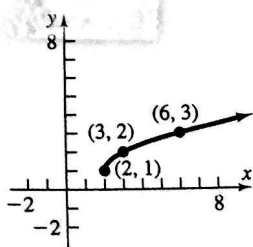
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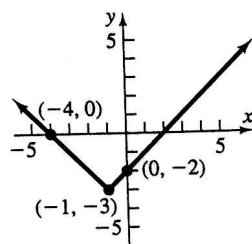
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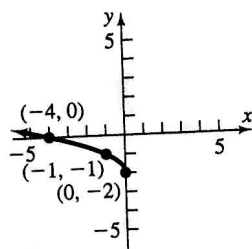
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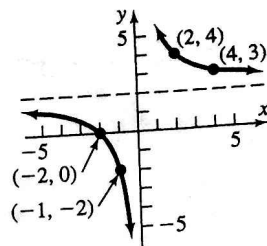
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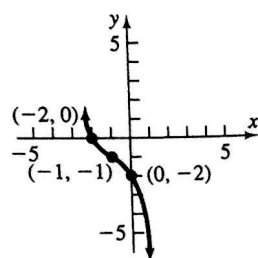
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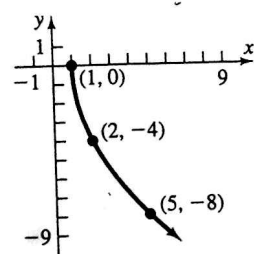
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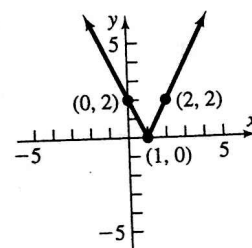
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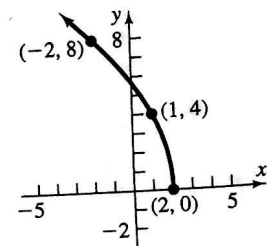
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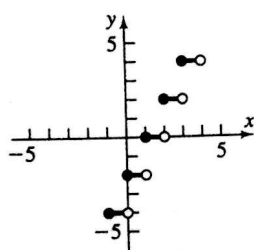
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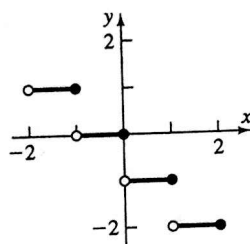
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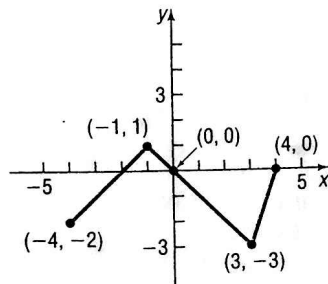
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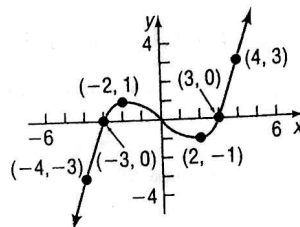
In Problems 27 and 28, use the graph of the function f to find:

- The domain and the range of f
- The intervals on which f is increasing, decreasing, or constant
- The local minima and local maxima
- Whether the graph is symmetric with respect to the x -axis, the y -axis, or the origin
- Whether the function is even, odd, or neither
- The intercepts, if any

27.



28.



In Problems 29–36, determine (algebraically) whether the given function is even, odd, or neither.

29. $f(x) = x^3 - 4x$ *odd* 30. $g(x) = \frac{4+x^2}{1+x^4}$ *even* 31. $h(x) = \frac{1}{x^4} + \frac{1}{x^2} + 1$ *even* 32. $F(x) = \sqrt{1-x^3}$ *Neither*
33. $G(x) = 1 - x + x^3$ *Neither* 34. $H(x) = 1 + x + x^2$ *Neither* 35. $f(x) = \frac{x}{1+x^2}$ *odd* 36. $g(x) = \frac{1+x^2}{x^3}$ *odd*

In Problems 37–40, use a graphing utility to graph each function over the indicated interval. Approximate any local maxima and local minima. Determine where the function is increasing and where it is decreasing.

37. $f(x) = 2x^3 - 5x + 1$ $(-3, 3)$ 38. $f(x) = -x^3 + 3x - 5$ $(-3, 3)$
39. $f(x) = 2x^4 - 5x^3 + 2x + 1$ $(-2, 3)$ 40. $f(x) = -x^4 + 3x^3 - 4x + 3$ $(-2, 3)$

In Problems 41 and 42, find the average rate of change of f :

- (a) From 1 to 2 (b) From 0 to 1 (c) From 2 to 4

41. $f(x) = 8x^2 - x$

42. $f(x) = 2x^3 + x$

In Problems 43–46, find the average rate of change from 2 to x for each function f . Be sure to simplify.

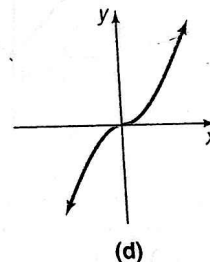
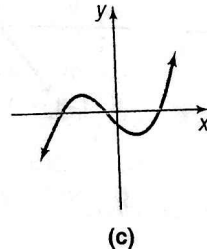
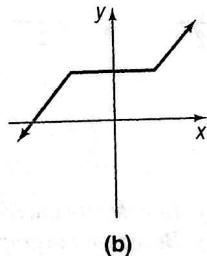
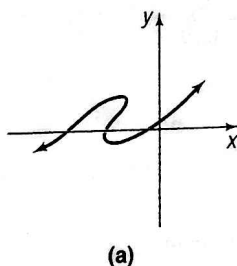
43. $f(x) = 2 - 5x$

44. $f(x) = 2x^2 + 7$

45. $f(x) = 3x - 4x^2$

46. $f(x) = x^2 - 3x + 2$

47. Tell which of the following are graphs of functions.



In Problems 48–50, sketch the graph of each function. Be sure to label at least three points.

48. $f(x) = |x|$

49. $f(x) = \sqrt[3]{x}$

50. $f(x) = \sqrt{x}$

In Problems 51–62, graph each function using the techniques of shifting, compressing or stretching, and reflections. Identify any intercepts on the graph. State the domain and, based on the graph, find the range.

51. $F(x) = |x| - 4$

52. $f(x) = |x| + 4$

53. $g(x) = -2|x|$

54. $g(x) = \frac{1}{2}|x|$

55. $h(x) = \sqrt{x-1}$

56. $h(x) = \sqrt{x} - 1$

57. $f(x) = \sqrt{1-x}$

58. $f(x) = -\sqrt{x+3}$

59. $h(x) = (x-1)^2 + 2$

60. $h(x) = (x+2)^2 - 3$

61. $g(x) = 3(x-1)^3 + 1$

62. $g(x) = -2(x+2)^3 - 8$

In Problems 17–24, sketch the graph of each function. Be sure to label at least three points on the graph.

17. $f(x) = x$

18. $f(x) = x^2$

19. $f(x) = x^3$

20. $f(x) = \sqrt{x}$

21. $f(x) = \frac{1}{x}$

22. $f(x) = |x|$

23. $f(x) = \sqrt[3]{x}$

24. $f(x) = 3$

25. If $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

find: (a) $f(-2)$ (b) $f(0)$ (c) $f(2)$

26. If $f(x) = \begin{cases} x^3 & \text{if } x < 0 \\ 3x + 2 & \text{if } x \geq 0 \end{cases}$

find: (a) $f(-1)$ (b) $f(0)$ (c) $f(1)$

27. If $f(x) = \text{int}(2x)$, find: (a) $f(1.2)$ (b) $f(1.6)$ (c) $f(-1.8)$

28. If $f(x) = \text{int}\left(\frac{x}{2}\right)$, find: (a) $f(1.2)$ (b) $f(1.6)$ (c) $f(-1.8)$

In Problems 29–40:

(a) Find the domain of each function.

(c) Graph each function.

(b) Locate any intercepts.

(d) Based on the graph, find the range.

29. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

31. $f(x) = \begin{cases} -2x + 3 & x < 1 \\ 3x - 2 & x \geq 1 \end{cases}$

33. $f(x) = \begin{cases} x + 3 & -2 \leq x < 1 \\ 5 & x = 1 \\ -x + 2 & x > 1 \end{cases}$

35. $f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

37. $f(x) = \begin{cases} |x| & \text{if } -2 \leq x < 0 \\ 1 & \text{if } x = 0 \\ x^3 & \text{if } x > 0 \end{cases}$

39. $f(x) = 2 \text{int}(x)$

30. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

32. $f(x) = \begin{cases} x + 3 & x < -2 \\ -2x - 3 & x \geq -2 \end{cases}$

34. $f(x) = \begin{cases} 2x + 5 & -3 \leq x < 0 \\ -3 & x = 0 \\ -5x & x > 0 \end{cases}$

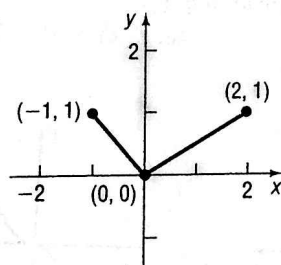
36. $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$

38. $f(x) = \begin{cases} 3 + x & \text{if } -3 \leq x < 0 \\ 3 & \text{if } x = 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$

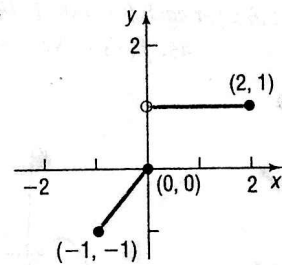
40. $f(x) = \text{int}(2x)$

In Problems 41–44, the graph of a piecewise-defined function is given. Write a definition for each function.

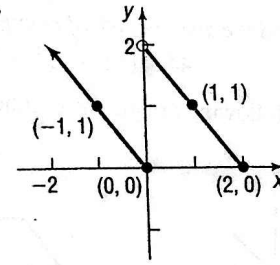
41.



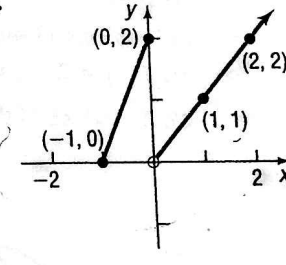
42.



43.



44.



In Problems 63–66:

(a) Find the domain of each function.

(c) Graph each function.

(b) Locate any intercepts.

(d) Based on the graph, find the range.

63. $f(x) = \begin{cases} 3x & -2 < x \leq 1 \\ x + 1 & x > 1 \end{cases}$

64. $f(x) = \begin{cases} x - 1 & -3 < x < 0 \\ 3x - 1 & x \geq 0 \end{cases}$

65. $f(x) = \begin{cases} x & -4 \leq x < 0 \\ 1 & x = 0 \\ 3x & x > 0 \end{cases}$

66. $f(x) = \begin{cases} x^2 & -2 \leq x \leq 2 \\ 2x - 1 & x > 2 \end{cases}$

67. Given that f is a linear function, $f(4) = -5$ and $f(0) = 3$, write the equation that defines f .

68. Given that g is a linear function, with slope $= -4$ and $g(-2) = 2$, write the equation that defines g .

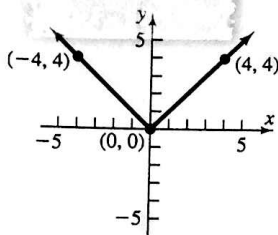
69. A function f is defined by

$$f(x) = \frac{Ax + 5}{6x - 2}$$

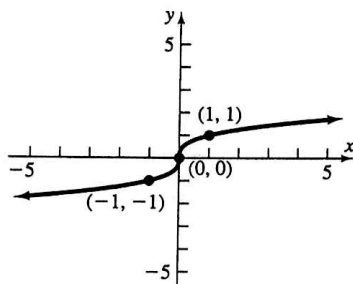
70. A function g is defined by

$$g(x) = \frac{A}{x} + \frac{8}{x^2}$$

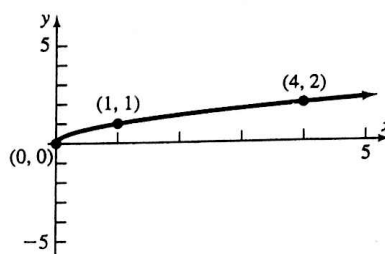
48.



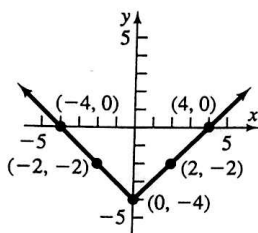
49.



50.

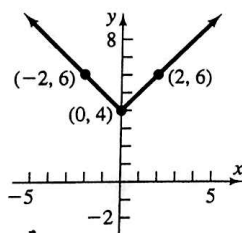


51.



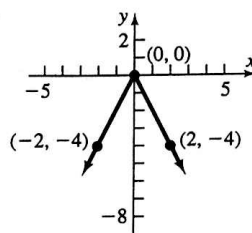
Intercepts: $(-4, 0), (4, 0), (0, -4)$
 Domain: all real numbers
 Range: $\{y|y \geq -4\}$ or $[-4, \infty)$

52.



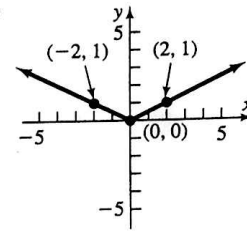
Intercept: $(0, 4)$
 Domain: all real numbers
 Range: $\{y|y \geq 4\}$ or $[4, \infty)$

53.



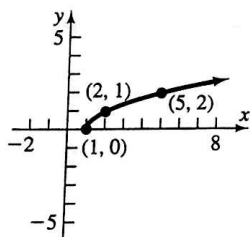
Intercept: $(0, 0)$
 Domain: all real numbers
 Range: $\{y|y \leq 0\}$ or $(-\infty, 0]$

54.



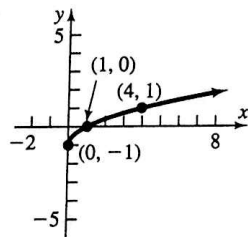
Intercept: $(0, 0)$
 Domain: all real numbers
 Range: $\{y|y \geq 0\}$ or $[0, \infty)$

55.



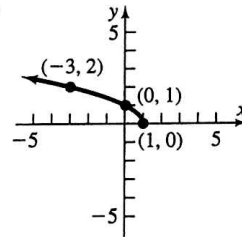
Intercept: $(1, 0)$
 Domain: $\{x|x \geq 1\}$ or $[1, \infty)$
 Range: $\{y|y \geq 0\}$ or $[0, \infty)$

56.



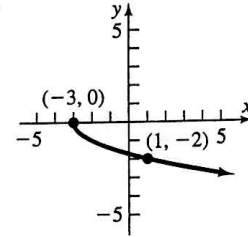
Intercepts: $(0, -1), (1, 0)$
 Domain: $\{x|x \geq 0\}$ or $[0, \infty)$
 Range: $\{y|y \geq -1\}$ or $[-1, \infty)$

57.



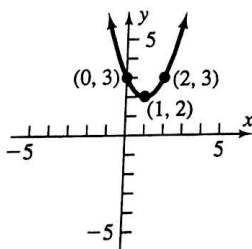
Intercepts: $(0, 1), (1, 0)$
 Domain: $\{x|x \leq 1\}$ or $(-\infty, 1]$
 Range: $\{y|y \geq 0\}$ or $[0, \infty)$

58.



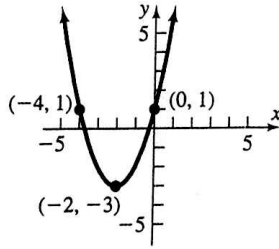
Intercepts: $(-3, 0), (0, -\sqrt{3})$
 Domain: $\{x|x \geq -3\}$ or $[-3, \infty)$
 Range: $\{y|y \leq 0\}$ or $(-\infty, 0]$

59.



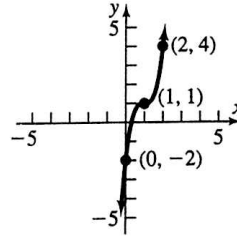
Intercept: $(0, 3)$
 Domain: all real numbers
 Range: $\{y|y \geq 2\}$ or $[2, \infty)$

60.



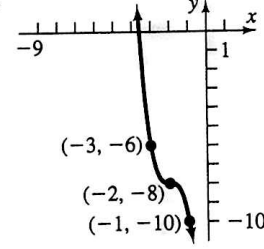
Intercepts: $(-2 - \sqrt{3}, 0), (-2 + \sqrt{3}, 0), (0, 1)$
 Domain: all real numbers
 Range: $\{y|y \geq -3\}$ or $[-3, \infty)$

61.



Intercepts: $(0, -2), (1 + \frac{\sqrt[3]{-9}}{3}, 0)$ or about $(0.3, 0)$
 Domain: all real numbers
 Range: all real numbers

62.

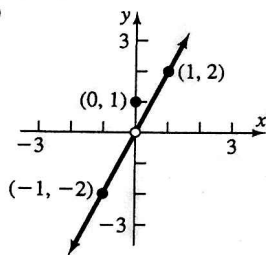


Intercepts: $(0, -24), (-2 + \sqrt[3]{-4}, 0)$ or about $(-3.6, 0)$
 Domain: all real numbers
 Range: all real numbers

29. (a) All real numbers

(b) (0, 1)

(c)

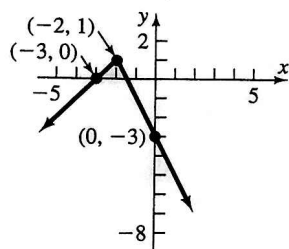


(d) $\{y|y \neq 0\}; (-\infty, 0) \text{ or } (0, \infty)$

32. (a) All real numbers

(b) $(-3, 0), (-\frac{3}{2}, 0), (0, -3)$

(c)

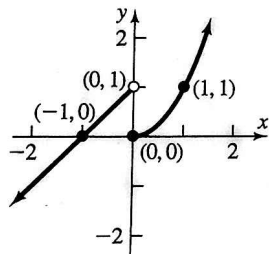


(d) $\{y|y \leq 1\}; (-\infty, 1]$

35. (a) All real numbers

(b) $(-1, 0), (0, 0)$

(c)

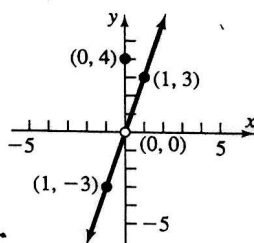


(d) All real numbers

30. (a) All real numbers

(b) (0, 4)

(c)

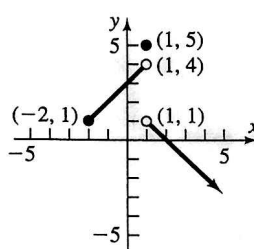


(d) $\{y|y \neq 0\}; (-\infty, 0) \text{ or } (0, \infty)$

33. (a) $\{x|x \geq -2\}; [-2, \infty)$

(b) (0, 3), (2, 0)

(c)

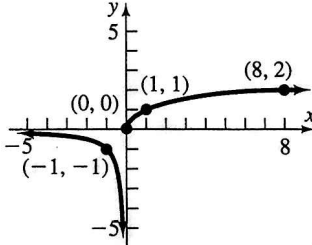


(d) $\{y|y < 4, y = 5\}; (-\infty, 4) \text{ or } [5]$

36. (a) All real numbers

(b) (0, 0)

(c)

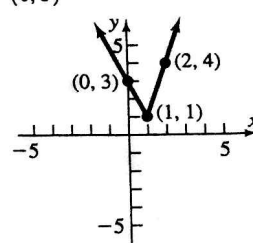


(d) All real numbers

31. (a) All real numbers

(b) (0, 3)

(c)

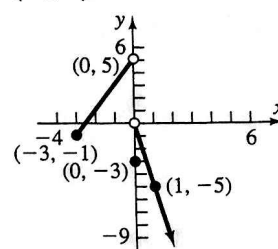


(d) $\{y|y \geq 1\}; [1, \infty)$

34. (a) $\{x|x \geq -3\}; [-3, \infty)$

(b) $(-\frac{5}{2}, 0), (0, -3)$

(c)

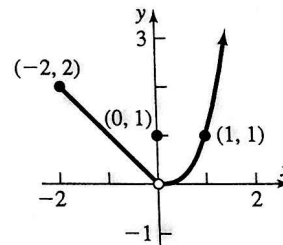


(d) $\{y|y < 5\}; (-\infty, 5)$

37. (a) $\{x|x \geq -2\}; [-2, \infty)$

(b) (0, 1)

(c)

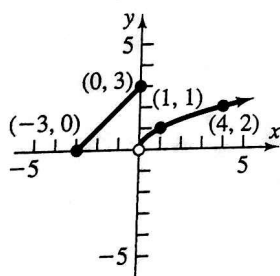


(d) $\{y|y > 0\}; (0, \infty)$

38. (a) $\{x|x \geq -3\}; [-3, \infty)$

(b) $(-3, 0), (0, 3)$

(c)

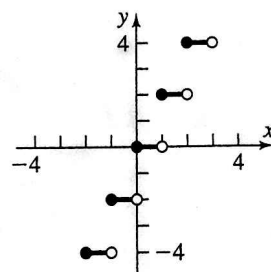


(d) $\{y|y \geq 0\}; [0, \infty)$

39. (a) All real numbers

(b) $(x, 0) \text{ for } 0 \leq x < 1$

(c)

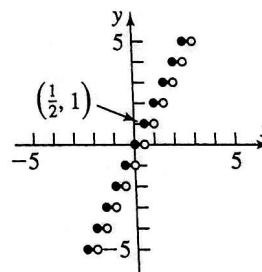


(d) Set of even integers

40. (a) All real numbers

(b) $(x, 0) \text{ for } 0 \leq x < \frac{1}{2}$

(c)



(d) Set of integers

41. $f(x) = \begin{cases} -x & \text{if } -1 \leq x \leq 0 \\ \frac{1}{2}x & \text{if } 0 < x \leq 2 \end{cases}$ (Other answers are possible.)

42. $f(x) = \begin{cases} x & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 2 \end{cases}$ (Other answers are possible.)

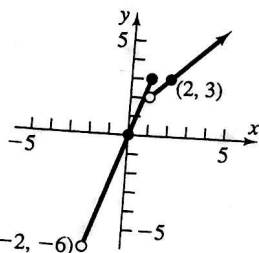
43. $f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ -x + 2 & \text{if } 0 < x \leq 2 \end{cases}$ (Other answers are possible.)

44. $f(x) = \begin{cases} 2x + 2 & \text{if } -1 \leq x \leq 0 \\ x & \text{if } x > 0 \end{cases}$ (Other answers are possible.)

63. (a) $\{x|x > -2\}; (-2, \infty)$

(b) $(0, 0)$

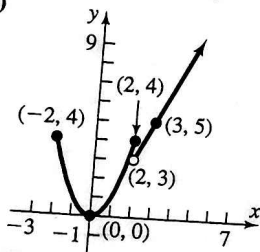
(c)



(d) $\{y|y > -6\}; (-6, \infty)$

66. (a) $\{x|x \geq -2\}; [-2, \infty)$

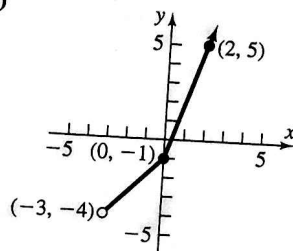
(c)



64. (a) $\{x|x > -3\}; (-3, \infty)$

(b) $(0, -1), \left(\frac{1}{3}, 0\right)$

(c)



(d) $\{y|y > -4\}; (-4, \infty)$

(b) $(0, 0)$

(d) $\{y|y \geq 0\}; [0, \infty)$

67. $f(x) = -2x + 3$

68. $g(x) = -4x - 6$

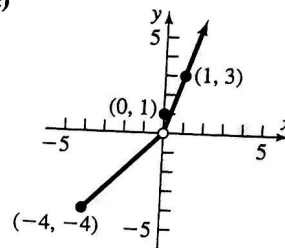
69. $A = 11$

70. $A = 8$

65. (a) $\{x|x \geq -4\}; [-4, \infty)$

(b) $(0, 1)$

(c)



(d) $\{y|y \geq -4\}; [-4, \infty)$